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# Value of DNA Tests: A Decision Perspective

**ABSTRACT:** Before a Court of Law testifying in DNA-evidence cases, scientists are often challenged with the idea that the more markers (*loci*) the better, i.e., why does the scientist not use 16 or more markers? This paper introduces a new perspective, decision analysis, to deal with the problem of the number of markers to type in a criminal context. The decision-making process, which plays a key role in the routine work of a forensic scientist, consists of the rational choice, given personal objectives, between two or more possible outcomes when the consequences of the choice are uncertain. Simulated results support the hypothesis that analytical added value does not increase with the number of markers.

**KEYWORDS:** forensic science, decision analysis, DNA evidence, evidence evaluation, interpretation, utility theory

There have been several efforts to clarify the approach to interpreting DNA evidence (e.g., Evett and Weir (1)). Despite these efforts, there are still important questions for the forensic scientist. One is of extreme relevance: *how many markers must be typed for forensic purposes to compare the crime sample with the suspect's material?* The question appears to be a simple one because, for practical reasons, the scientist uses commercial kits offering, for example, results of 16 markers simultaneously. But this question is being asked for a more fundamental reason and the answer has repercussions for other forensic disciplines. To answer this question, it is crucial to be familiar with the added value offered by each supplementary marker typed. Some authors (2) have noticed that in Court, there are often suggestions that the more markers the better, i.e. why did the scientist not use 10, 11, 12, or more markers?

It is also not uncommon to hear about the need to increase the number of markers through the development of new *n-locus STR profiling systems* (with $n>16$). One reason is that in case it is necessary, the scientist will achieve a greater discrimination. Moreover, it is claimed that if the size of a database used to select a potential suspect increases, the number of markers analyzed should increase as well to guarantee discrimination.

The fundamental questions are still open: how is one to decide the number of markers to type? How can the scientist be sure that results from, say, 20 markers are enough for forensic purposes? What is the meaning of "forensic purposes"?

As mentioned by Triggs and Buckleton (3) (p. 108):

> The DNA technology is so powerful that it can accommodate inaccurate interpretation procedures and still have little chance of leading to a false conclusion. The potential power and the technology has lead to recommendations that appear to be a combination of poor logic and a reliance on the technology to give the correct conclusion regardless of the lack of rigorous attention to principles of interpretation.

Solid logic should be used. This should be done by using sound mathematical and population-genetic principles. Therefore, studies describing the validation of STR systems have been performed. These studies have all found that STR multiplexes have a high power of discrimination. They have shown that a large majority of cases meet required levels of discrimination (e.g., Thomson et al. (4)). Further STR *loci* are available to allow additional testing in the small number of cases requiring it. It has been advocated that a simple three- or four-locus multiplex can easily be incorporated into the testing routine.

> The panel of 16 markers forms the basis of a highly informative and convenient system for investigation of parentage and other claimed relationships. [. . .] In terms of the ability to exclude non-parents and to provide a high level of certainty in cases of non-exclusion [. . .]. (Thomson et al. (5) p. 133)

This paper introduces a new perspective, decision analysis, to deal with the problem at hand. The decision-making process, which plays a key role in the routine work of a forensic scientist, consists of the rational choice, given personal objectives, between two or more possible outcomes when the consequences of the choice are uncertain (6).

Decision analysis helps the scientist to better understand the problem he is faced with and to make clearer and more consistent decisions. The aim of this paper is to propose an approach that allows one to think systematically about a problem, while offering more structure and guidance in answering such questions as: "what conclusions can be drawn from the available evidence?" and "why is the chosen action appropriate?"

This paper approaches these points through the use of an example involving DNA evidence in forensic science to justify the number of markers to type in a criminal context. Graphical models, such as influence diagrams (also called Bayesian decision networks), are also introduced to deal with the problem at hand.

## Decision Analysis

Should you buy a Lotto ticket? Should you type a number, $n$, of DNA markers? In each case, you decide upon an action. Hacking (7) describes the model needed when we are uncertain not only

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about what will happen or what is true but also about what action to take. Therefore, decisions need more than probabilities.

> Actions have consequences. You waste a dollar (or perhaps win a fortune). You take no action at all, which in itself is an action. Some consequences are desirable; some are not. Suppose you can represent the cost or benefit of a possible consequence by a number—an amount in dollars, perhaps. Call that number the *utility* of the consequence. Suppose you can also represent the probability of each possible consequence of an action by a number.
> In making a decision, we want to assess the relative merits of each possible action. There is a simple way to combine probabilities and utilities in order to evaluate possible actions: multiply them together. Multiply the probability by the utility of each consequence of an action and then add the results for each possible consequence of the action. The result of this calculation is called the *expected value* of an action. (Hacking (7) p. 79–80)

As outlined above, faced by a decision problem, it is of extreme importance to identify the elements of the situation under investigation. There are four relevant elements (8):

1. *Objectives*. They refer to important matters to the decision-maker. In what follows, the objective is the optimization of the analytical strategy. We may say that the objective is to type the optimal number of markers to support the hypothesis, i.e., that a given suspect is the source of a bloodstain (prosecutor's perspective) or that an unrelated person, randomly selected, is the source of the stain (defense's perspective).
2. *Decision to Make*. To identify the immediate decision to make is a critical step in understanding a difficult decision problem. In identifying the central decision, it is important to think about possible alternatives. This paper approaches the problem of deciding the number of genetic markers to type, so the possible alternatives are given by the increasing number of markers to type ($1, 2, \ldots, m$).
3. *States of Nature*. Decision problems can be complicated because of uncertainty about what the future holds. The possible things that can happen in the resolution of state of nature are called *outcomes*. In the specific case, the state of nature will be represented by apportionments of likelihood ratio.
4. *Consequences*. Once a decision is made and the state of nature occurs, then the occurrence of the event will produce a definite result that can be foreseen with certainty. The measure of the extent of objective achievement is called *utility*, or preference for one of the consequences.

Let us introduce some useful notation. First, let us consider an exhaustive list of actions that are available: $d_1, d_2, \ldots, d_m \in \Delta$. It is convenient to make the requirement of exclusivity: only one of the decisions can be selected.

Second, a list of $n$ exclusive and exhaustive states of nature is needed: $\theta_1, \theta_2, \ldots, \theta_n \in \Omega$. It is possible to measure the uncertainty of the events using a suitable probability distribution, $P$, over $\Omega$. Therefore, each alternative is associated with a probability distribution and a choice among probability distributions.

The combination of decision $d_i$ with state of nature $\theta_j$ will result in a foreseeable consequence. This consequence will be written as $C_{ij}$. Varying $d_i$, $i = 1, \ldots, m$, and $\theta_j$, $j = 1, \ldots, n$, a space of consequences is obtained. Consequences are defined in such a way that it is possible for them to be ranked as "best" for the first, and "worst" for the last. With this particular ranking, it is not immediately obvious which decision should be taken. It follows that the next task is to provide something more than just a ranking. In order to do this, a standard is introduced and a coherent comparison with this standard provides a numerical assessment.

Let us assume that $C$ is the best consequence and $c$ is the worst of them, $C$ and $c$ being a reference pair of highly desirable and highly undesirable consequences, respectively. It follows that any consequence $C_{ij}$ may be compared unfavorably with C and favorably with $c$. Associated with any consequence $C_{ij}$ is a unique number, $u \in (0,1)$, such that $C_{ij}$ is just as desirable a probability $u$ of $C$ as $1 - u$ of $c$. The number associated with $C_{ij}$ will be denoted $u(C_{ij})$, or equally $u(d, \theta_j)$, and will be called the *utility* of $C_{ij}$.

Utility is a measure of the desirability of the consequences of a course of action that applies to decision-making under risk. Utility is a probability: it is by definition the probability of obtaining the best consequence (9). Note that consequences will be valued differently by different people. The value inserted for the utilities is by no means correct or that any other values are necessarily wrong. They represent decision-makers' individual preferences and may be modified by the individuals. The only inviolate feature is their coherence (9).

To be a rational decision-maker, one must choose the action offering the highest probability of obtaining the best consequence. If decision $d_i$ is taken and if state $\theta_j$ occurs, the probability of obtaining the consequence $C$ is

$$\Pr(C|d_i, \theta_j) = u(C_{ij}).$$

A decision problem is solved by maximizing expected utility. This rule is known as the *rule of the maximization of the expected utility*. The *expected utility* of decision $d_i$ gives a numerical value to the probability of obtaining the best consequence if decision $d_i$ is taken:

$$E(U|d_i) = \sum_{j=1}^{n} \Pr(C|d_i, \theta_j) \Pr(\theta_j|d_i) = \sum_{j=1}^{n} u(C_{ij}) \Pr(\theta_j|d_i). \quad (1)$$

The numerical order of expected utilities of actions preserves the decision-maker's preference order among these actions.

Note that probabilities of the states of nature depend on the decision adopted. In many decision problems, there is not this same dependency, and so $\Pr(\theta_j)$ is written, omitting the decision (see the example presented in Taroni (6)). In practice, they can, but this complication does not affect the method. In fact, in "extending the conversation" (10) from $C$ to include the $\theta$s, we should have to combine $\Pr(C|d_i, \theta_j)$ with $\Pr(\theta_j|d_i)$, and not only with $\Pr(\theta_j)$.

An application of this theory in a judicial context has been suggested by Lindley (11). Examples in forensic science appear in Taroni (6,12).

### *Likelihood Ratios as States of Nature*

The aim of this paper is to estimate the value of the information given by each new marker typed in a DNA context. It is important to assess after how many markers it is reasonable to believe that a new one will not increase the total information available. This has to be done for cases where the evidence is assessed under two competing hypotheses: (1) the suspect is the source of the recovered stain; and (2) another person (unrelated to the suspect) is the

TABLE 1—*Apportionments of likelihood ratio and states of nature.*

| V | States of Nature |
|---|---|
| [0,1) | $\theta_1$ |
| [1,10) | $\theta_2$ |
| [10,100) | $\theta_3$ |
| [100,1000) | $\theta_4$ |
| [1000,10,000) | $\theta_5$ |
| [10,000, 100,000) | $\theta_6$ |
| [100,000, 1,000,000) | $\theta_7$ |
| [1,000,000, 10,000,000) | $\theta_8$ |
| ≥10,000,000 | $\theta_9$ |

source of this stain, so that the match between the profile $E_c$ of the crime sample and the profile $E_s$ of the suspect is accidental (a match due to the chance alone).

This is in accordance with a widely accepted foundation that a likelihood ratio, $V$, represents a unified measure for the value of scientific evidence (13). The likelihood ratio in a common situation involving DNA, where there are both profiles $E_c$ and $E_s$ and where $I$ represents the background information, can be expressed as

$$V = \frac{\Pr(E_c|E_s, H_p, I)}{\Pr(E_c|E_s, H_d, I)},$$

where propositions, at the source level (14), are $H_p$: the suspect is the source of the stain, $H_d$: another person, unrelated to the suspect, is the source of the stain.

In a pre-assessment perspective, where the scientist would like to offer an answer to the question of obtaining a value of evidence supporting the proposition $H_p$ or $H_d$, the estimation of theoretical distributions of likelihood ratios is fundamental (15).

States of nature have been defined as apportionments of likelihood ratio in adequacy to verbal scales proposed in forensic literature (e.g., Evett et al. (16,17)). The apportionments of likelihood ratio and the states of nature are given in Table 1. Note that $\theta_8$ represents the last interval considered because studies have shown that the "independence assumptions" between *loci* in a second generation multiplex are sufficiently reliable to infer probabilities that are of the order of one in tens of millions (18). This offers likelihood ratios of the order of tens of millions. Smaller probability values (greater likelihood ratios), although not necessarily wrong, are without any real meaning; it might be argued that even this apparently cautious figure lies beyond the range of numbers that can be supported by conventional independent testing ((19) pp. 347–48).

An assessment of probabilities for each state of nature is needed. From a practical point of view, allele frequencies (at different *loci*) from a selected population database are chosen. It is assumed that these frequencies can be used to generate new databases through simulation techniques as suggested by Triggs and Buckleton (3).

Two databases of a large number of pairs of individuals are generated. For each pair, the first genotype represents characteristics of the source of the recovered stain, and the second is the suspect's genotype. The first database is composed of pairs of individuals having the same genotype at each locus: under proposition $H_p$, the suspect and the source of the stain are the same person.

The second database is composed of pairs of unrelated individuals. For each pair, the two genotypes generally do not match. A possible match is due to chance alone: different individuals will match at some *loci*, but the large majority of *loci* will not.

The dimension of the databases is fixed to be 100,000 pairs of individuals. Simulations have shown that this dimension is sufficient to estimate the distributions of the likelihood ratios.

For each given pair of individuals in the first database, a likelihood ratio is estimated. In this way, it is possible to estimate the theoretical distribution of $V$ under $H_p$. The same procedure is performed for pairs of individuals coming from the second database (unrelated individuals), and so the distribution of $V$ under $H_d$ can be estimated.

A co-ancestry coefficient, $F_{ST}$, set to be 0.01, is used in simulations to take into account the effect of sub-populations in assessing random match probabilities as suggested by Balding and Nichols (20). Tables 2 and 3 show the probabilities of each state of nature calculated under propositions $H_p$ and $H_d$, respectively. Note that probabilities of the states of nature depend on the number of markers typed, $d_i$, and are developed using "extension the conversation":

$$\Pr(\theta_j|d_i) = \Pr(\theta_j|d_i, H_p)\Pr(H_p) + \Pr(\theta_j|d_i, H_d)\Pr(H_d), \qquad (2)$$

where $\Pr(H_p)$ and $\Pr(H_d)$ represent the prior probabilities that the suspect is, or is not, the source of the recovered material, respectively.

The values in Table 2 show that if $H_p$ is true ($\Pr(H_p) = 1$), so that the suspect is the donor of the stain, the probabilities of obtaining high values of $V$ are very high. In particular, with more

TABLE 2—*Probabilities of the states of nature under proposition* $H_p$.

| | $\Pr(\theta_1)$ | $\Pr(\theta_2)$ | $\Pr(\theta_3)$ | $\Pr(\theta_4)$ | $\Pr(\theta_5)$ | $\Pr(\theta_6)$ | $\Pr(\theta_7)$ | $\Pr(\theta_8)$ | $\Pr(\theta_9)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 locus | 0 | 0.49492 | 0.0087 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 loci | 0 | 0 | 0.31661 | 0.65125 | 0.03165 | 0.00049 | 0 | 0 | 0 |
| 3 loci | 0 | 0 | 0 | 0.11859 | 0.69493 | 0.17566 | 0.01057 | 0.00025 | 0 |
| 4 loci | 0 | 0 | 0 | 0 | 0.00230 | 0.4580 | 0.456820 | 0.07778 | 0.0051 |
| 5 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0.1034 | 0.54055 | 0.35605 |
| 6 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0.00067 | 0.13839 | 0.86094 |
| 7 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.0023 | 0.99797 |
| 8 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 9 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 10 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 11 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 12 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 13 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 14 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 15 loci | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

TABLE 3—*Probabilities of the states of nature under proposition $H_d$ (unrelated).*

| | $Pr(\theta_1)$ | $Pr(\theta_2)$ | $Pr(\theta_3)$ | $Pr(\theta_4)$ | $Pr(\theta_5)$ | $Pr(\theta_6)$ | $Pr(\theta_7)$ | $Pr(\theta_8)$ | $Pr(\theta_9)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 locus | 0.91109 | 0.06396 | 0.02512 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 loci | 0.99215 | 0 | 0.00441 | 0.00344 | 0 | 0 | 0 | 0 | 0 |
| 3 loci | 0.99965 | 0 | 0 | 0.0001 | 0.00024 | 0.00001 | 0 | 0 | 0 |
| 4 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 8 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 9 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 10 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 11 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 12 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 13 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 14 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

than seven markers typed, the likelihood ratios are always greater than 10,000,000.

The values in Table 3 show greater probabilities for low-likelihood ratio apportionments, notably in states $\theta_1$–$\theta_4$. In fact, if $H_d$ is true ($Pr(H_d) = 1$), so that the suspect is not the source of the stain, the probability of obtaining a value of $V < 1$ is greater than in the previous situation ($Pr(H_p) = 1$).

There is clearly an overlap between the two distributions. The overlap decreases and finally disappears as the number of markers typed increases. Note that the values presented in Tables 2 and 3 provide the answer to the pre-assessment question of obtaining a likelihood ratio value supporting either the proposition $H_p$ or $H_d$. The values of $V$ show that an informed result can be obtained.

Such simulations allow the scientist to detect how many markers should be typed so that any further marker does not bring any more valuable information. From Table 2, it can be observed that, if the subject is the source of the stain, there is no need to type more than eight markers. Conversely, from Table 3, it can be observed that, if the subject is not the source of the stain and an unrelated person is, then there is no reasonable need to type more than four markers.

### *Related Persons as a Relevant Population*

Consider the possibility that there might be pairs of individuals who have the same profile for some marker. In this case, a match can be misleading and so a larger number of markers is recommended. Therefore, instead of using a database of pairs of unrelated individuals characterizing the relevant population under proposition $H_d$, the use of a database of pairs of individuals who will likely have common alleles appears to be more appropriate. A database with these characteristics is constituted by pairs of full siblings; this represents an extremely conservative situation. The probabilities of the states of nature are then computed as illustrated above and given in Table 4. It can be observed that once 14 markers have been typed, any further marker will not bring more information, as the probability of finding a nonmatch approaches 1.

The results presented in Table 4 are in agreement with forensic literature (e.g., Evett et al. (19)) in which the great potential of the 10-*loci* system has been studied when close relatives are considered. The probability that two brothers would have the same six *loci* profile is roughly 1 in 500 (0.00182 for $Pr(\theta_9)$), whereas with 10 *loci*, this probability is in the order of 1 in 20,000 (0.00005 for $Pr(\theta_9)$).

### *Utility as a Measure for the Value of Information*

In the previous section, simulations were performed to investigate how the number of markers typed affects the probability of the states of nature. Utility theory can now be implemented to quantify the added value of information offered by each new marker typed.

Utilities are quantified using the standard presented in the Section entitled "Decision analysis." It is reasonable to believe that a prosecutor, a defense lawyer, and a judge have different objec-

TABLE 4—*Probabilities of the states of nature under proposition $H_d$ (full siblings).*

| | $Pr(\theta_1)$ | $Pr(\theta_2)$ | $Pr(\theta_3)$ | $Pr(\theta_4)$ | $Pr(\theta_5)$ | $Pr(\theta_6)$ | $Pr(\theta_7)$ | $Pr(\theta_8)$ | $Pr(\theta_9)$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 locus | 0.61159 | 0.21102 | 0.17509 | 0.0023 | 0 | 0 | 0 | 0 | 0 |
| 2 loci | 0.85143 | 0 | 0.05345 | 0.0917 | 0.00338 | 0 | 0 | 0 | 0 |
| 3 loci | 0.94866 | 0 | 0 | 0.00774 | 0.0352 | 0.00809 | 0.00031 | 0 | 0 |
| 4 loci | 0.98308 | 0 | 0 | 0 | 0.0001 | 0.00814 | 0.00755 | 0.00108 | 0.0001 |
| 5 loci | 0.99467 | 0 | 0 | 0 | 0 | 0 | 0.00063 | 0.00295 | 0.00175 |
| 6 loci | 0.99784 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00034 | 0.00182 |
| 7 loci | 0.99906 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00094 |
| 8 loci | 0.99962 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00038 |
| 9 loci | 0.99989 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00011 |
| 10 loci | 0.99995 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00005 |
| 11 loci | 0.99998 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00002 |
| 12 loci | 0.99999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00001 |
| 13 loci | 0.99999 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0.00001 |
| 14 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 15 loci | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE 5—*Utility values for the states of nature given different objectives.*

| | $\Pr(\theta_1)$ | $\Pr(\theta_2)$ | $\Pr(\theta_3)$ | $\Pr(\theta_4)$ | $\Pr(\theta_5)$ | $\Pr(\theta_6)$ | $\Pr(\theta_7)$ | $\Pr(\theta_8)$ | $\Pr(\theta_9)$ |
|---|---|---|---|---|---|---|---|---|---|
| Prosecution | 0 | 0.1 | 0.1 | 0.2 | 0.3 | 0.5 | 0.7 | 0.9 | 1 |
| Defense | 1 | 0.9 | 0.7 | 0.5 | 0.3 | 0.2 | 0.1 | 0.1 | 0 |
| Judge | 1 | 0.9 | 0.8 | 0.5 | 0.4 | 0.5 | 0.8 | 0.9 | 1 |

tives, and so different utility functions. The prosecutor will probably assign high utility values to states of nature corresponding to high values of likelihood ratio. On the other hand, the defense lawyer will assign high utility values to the states of nature corresponding to low values of likelihood ratio. The judge will have a more neutral position as he must consider the possibility that the suspect might or might not be the source of the stain. Therefore, it is reasonable to think that he will assign high utility values to extreme states of nature corresponding to apportionments of likelihood ratio that strongly support both propositions. Purely for the sake of illustration, different utility values corresponding to the different objectives are proposed in Table 5.

Utilities, $u$, assigned to intermediate consequences are specified in answering the question: does the decision-maker prefer the intermediate consequence or does he prefer the best consequence with a probability set equal to $u$?

The value $u$ is the threshold of indifference. It is also assumed that the utility is constant for all possible decisions: $u(\theta_j, d_i) = u(\theta_j)$, $\forall\, j = 1, \ldots, n$, $\forall\, j = 1, \ldots, m$.

Consider the prosecution perspective and compute the expected utility of typing $i$ markers, $i = 1, \ldots, m$. The expected utility of decision $d_i$ will be

$$E(U|d_i) = \sum_{j=1}^{n} u(\text{Prosecution}, \theta_j) \cdot \Pr(\theta_j|d_i). \tag{3}$$

The prosecutor will generally assign a probability close to 1 to the proposition that the suspect is the source of the stain, $\Pr(H_p)$. The defense will do conversely, while the judge will generally have a neutral position. In Table 6, the expected utilities are computed for the prosecutor's, defense's and judge's perspectives, with prior probability, $\Pr(H_p)$, set equal to 1, 0, and 0.5, respectively.

Note that it is generally inadvisable to attach probabilities of zero or one to states of nature (Cromwell's rule, (9)), because a

TABLE 6—*Expected utility.*

| | Prosecution | | Defense | | Judge |
|---|---|---|---|---|---|
| | $\Pr(H_p) = 1$ | $\Pr(H_p) = 0.9$ | $\Pr(H_p) = 0$ | $\Pr(H_p) = 0.1$ | $\Pr(H_p) = 0.5$ |
| 1 locus | 0.1009 | 0.0917 | 0.9861 | 0.9672 | 0.9178 |
| 2 loci | 0.1717 | 0.1546 | 0.9970 | 0.9529 | 0.7946 |
| 3 loci | 0.3277 | 0.2949 | 0.9998 | 0.9302 | 0.7168 |
| 4 loci | 0.6246 | 0.5621 | 1 | 0.9146 | 0.8352 |
| 5 loci | 0.9149 | 0.8234 | 1 | 0.9064 | 0.9626 |
| 6 loci | 0.9860 | 0.8874 | 1 | 0.9014 | 0.9930 |
| 7 loci | 0.9998 | 0.8998 | 1 | 0.9 | 0.9999 |
| 8 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 9 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 10 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 11 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 12 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 13 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 14 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 15 loci | 1 | 0.9 | 1 | 0.9 | 1 |

TABLE 7—*Expected utility in cases involving related persons (full siblings).*

| | Prosecution | | Defense | | Judge |
|---|---|---|---|---|---|
| | $\Pr(H_p) = 1$ | $\Pr(H_p) = 0.9$ | $\Pr(H_p) = 0$ | $\Pr(H_p) = 0.1$ | $\Pr(H_p) = 0.5$ |
| 1 locus | 0.1009 | 0.0947 | 0.9252 | 0.9124 | 0.8949 |
| 2 loci | 0.1717 | 0.1570 | 0.9357 | 0.8978 | 0.7666 |
| 3 loci | 0.3277 | 0.2965 | 0.9647 | 0.8987 | 0.7023 |
| 4 loci | 0.6246 | 0.5632 | 0.9856 | 0.9016 | 0.8324 |
| 5 loci | 0.9149 | 0.8239 | 0.9950 | 0.9020 | 0.9624 |
| 6 loci | 0.9860 | 0.8876 | 0.9979 | 0.8995 | 0.9930 |
| 7 loci | 0.9998 | 0.8999 | 0.9991 | 0.8992 | 0.9999 |
| 8 loci | 1 | 0.9 | 0.9996 | 0.8997 | 1 |
| 9 loci | 1 | 0.9 | 0.9999 | 0.8999 | 1 |
| 10 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 11 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 12 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 13 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 14 loci | 1 | 0.9 | 1 | 0.9 | 1 |
| 15 loci | 1 | 0.9 | 1 | 0.9 | 1 |

decision-maker must always leave some room for doubt. Table 6 also presents the expected utilities for both the prosecutor and the defense, which are computed using $\Pr(H_p)$ equal to 0.9 and 0.1, respectively. Note that with $\Pr(H_p) = 0.1$, the expected utility for the defense decreases. In fact, if the defense is sure that the suspect is not the source of the stain, then the expected utility increases for each new marker typed as the probability of finding a nonmatch increases. Conversely, if there is a doubt that the suspect might be the source of the stain, it is reasonable to believe that high values of $V$ will be observed. Therefore, the defense will not obtain a larger expected utility in typing more markers. This explains why the utility decreases. The more the defense believes the suspect is the source of the stain, the less desirable it would be to type extra markers. A sensitivity analysis to the prior probabilities will be performed in the Section entitled "Sensitivity to prior probabilities."

The expected utility in cases involving related persons such as siblings is given in Table 7.

The expected utility is constant after a number, say $x$, of markers typed (see Tables 6 and 7). For the sake of illustration, let us consider the prosecutor's perspective in Table 6. The expected utility of decision $d_8$ is equal to the expected utility of the decision $d_{15}$. One might argue that as a decision problem is solved by maximizing the expected utility, there would be no difference in typing more then eight markers, because the same expected utility is obtained. Nevertheless, the aim here is to offer a tool to measure the benefit given by every new marker typed. The value of information is assessed using the *expected benefit,* which represents a difference in the expected utilities, the difference made by introducing new information. For example, the expected benefit obtained from marker $i$ is calculated by

$$EB(\text{Marker}_i) = E(U|d_i) - E(U|d_{i-1})\; i = 2, \ldots, m. \tag{4}$$

So, in order to make a decision based on an increasing number of markers to type, it is necessary to compute for every new marker a measure on how much is gained, in terms of utility. Values of information can be graphically represented using a percentage increment in terms of utility as depicted in Figs. 1 and 2. In particular, in Fig. 1 it can be observed that the second to the fifth markers bring considerable information. A less significant contribution is made by the sixth and the seventh markers, while successive markers do not add any further valuable information.

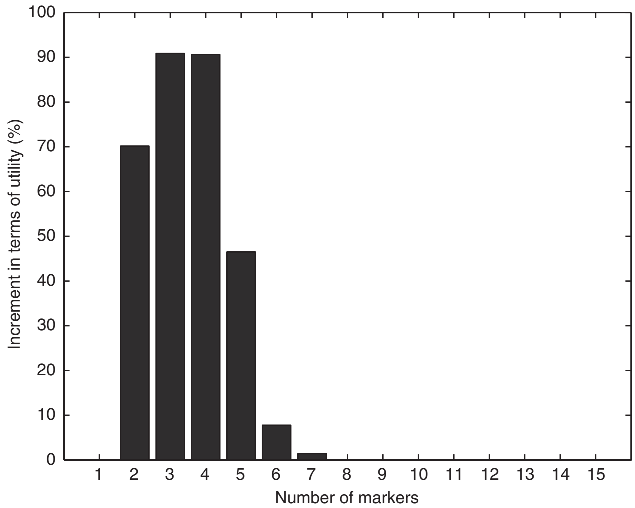


FIG. 1—*Expected benefit under the prosecution's proposition, $H_p$.*

In the same way, in Fig. 2, which represents a scenario involving full siblings as the relevant population, it can be noted that from the ninth analysis on, the contribution of extra markers is less than 0.05%. This means that every new marker typed will not increase the total amount of the information already available.

It can be said that a good decision is one that makes effective use of the information available to the decision-maker at the time the decision is made. The expected benefit represents a way of assessing the extent to which specific information will help the scientist to decide coherently among actions.

## Graphical Models and Sensitivity Analysis

Bayesian networks (BNs) provide a language of building blocks for constructing probability and decision models from modular components. They provide a valuable aid for representing relationships between characteristics in situations of uncertainty. They assist the user not only in describing a complex problem and communicating information about its structure but also in calculating the effect of knowing the truth of one proposition or one piece of evidence on the plausibility of others. BNs represent uncertainty and may be used for probabilistic inference.

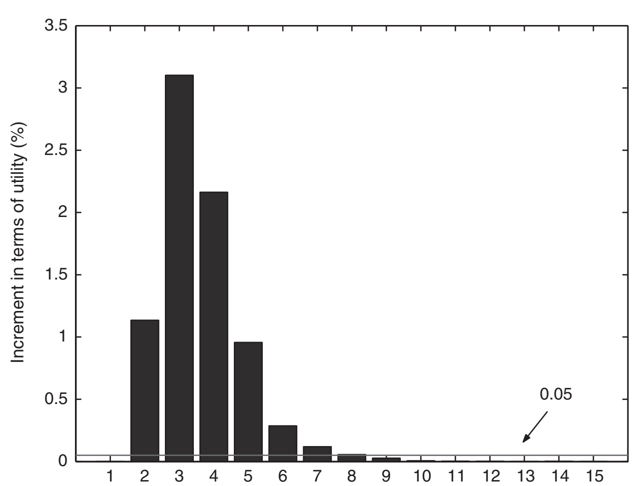


FIG. 2—*Expected benefit under the defense's proposition, $H_d$, in cases involving related persons (full siblings).*

Nodes and arcs are the main ingredients of a BN. Nodes represent a set of random variables. Each node is characterized by states describing the values that the corresponding variables can take. A set of directed arcs connects pairs of nodes representing the direct dependencies between variables. Nodes and arcs form a directed acyclic graph (21).

An extension of BNs provides the scientist with an aid to support decision-making. Adding an explicit representation of the decisions under consideration and the value (*utility*) of the resulting outcomes (the states that may result from a decision) gives *Bayesian decision networks*, BDNs, also called *influence diagrams*. These networks combine probabilistic reasoning with the utility theory to make decisions using the criteria of maximizing the expected utility.

An influence diagram consists of three types of nodes:

- the *chance* nodes represent random variables (as in BNs);
- the *decision* nodes represent decisions to make. The states of a decision node are the different actions that the decision-maker must choose between; and
- the *utility* nodes represent the decision-maker's utility function. They are characterized by utility tables specified for every outcome.

Figure 3 presents an influence diagram that can be implemented for the problem of interest. The decision node represents the decision about the number of markers to type, while the chance node represents the state of nature. The link between the decision node and the chance node allows the scientist to take into account the fact that the probability of each state of nature depends on the number of markers typed. So, the probability table associated with the chance node represents conditional probabilities $\Pr(\theta_j|d_i)$, $j = 1, \ldots, n$ and $i = 1, \ldots, m$.

An arc from the chance node to the utility node is added, as utilities depend on the state of nature. Note that utility values are also objective-dependent (i.e., prosecutor, defense, or judge). It can be noted that there is no parent (chance) node to the decision node (no arc pointing to this node). If information is known before making a decision, then an arc from this chance node to the decision node is necessary; this is not the case in the situation of interest. Note also that there is no link between the decision and the utility node. This is because it is assumed that utility values are constant whatever the number of markers typed.

Decision analysis is typically an iterative process: once the model is built, sensitivity analysis is performed. Sensitivity analysis is a general technique for studying the effects of modifications of the input parameters of a model on the output. This approach answers questions such as "if we make a slight change in one or more aspects of the model, does the optimal decision

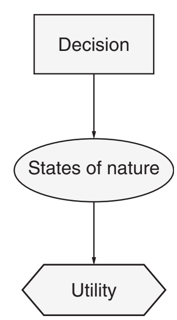


FIG. 3—*Influence diagram.*

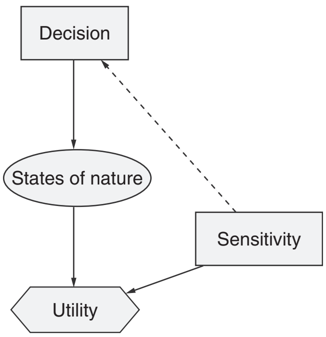


FIG. 4—*Influence diagram used to perform the sensitivity analysis on utilities.*

change?" (22). The main purpose is to obtain an idea about which aspects are determinant for the decision.

The simplest way to perform sensitivity analysis is to vary systematically one of the network parameters, keeping all others fixed. This analysis is termed "*one-way sensitivity analysis*." If several parameters are modified simultaneously, sensitivity analysis is called "*multi-way*" (examples are presented in Beidermann and Taroni (23)).

Attention is focused on two elements: the utility values and the prior probabilities of the main proposition, $H_p$. The two aspects are studied separately through "*one-way*" sensitivity analysis using the influence diagram built in GeNie (software available at http://www2.sis.pitt.edu/~genie/).

### *Sensitivity to Utility Values*

Utility values are, by definition, personal. They are always a choice of the decision-maker, and therefore they depend on his objective. Sensitivity analysis is performed to check how much the optimal strategy is affected by different assessments in terms of utility values. Figure 4 presents the influence diagram used to perform the sensitivity analysis on the utility values.

The probabilities for each state of nature are computed using Eq. (2), where $\Pr(\theta_j|d_i, H_p)$ and $\Pr(\theta_j|d_i, H_d)$ are as in Tables 2 and 3, while $\Pr(H_p)$ and $\Pr(H_d)$ are set to be equal to 1. The slope of the utility function has been modified for each objective studied: the prosecutor's, the defense lawyer's, and the judge's, respectively. In each situation, three different utility functions (cases (*a*), (*b*), (*c*)) have been proposed, keeping the original tendency unchanged (see Figs. 5, 6, and 7, left). Note that the second case, (*b*), represents the original utility function used in the Section entitled "Utility as a measure for the value of information."

The expected utilities obtained for the prosecutor are presented in Fig. 5 (right) and they show that the optimal number of markers to type could decrease until five *loci* under specific conditions, such as the situation (*c*).

For the defense lawyer, the optimal decision is not sensitive to utility values. This follows immediately from probabilities for the states of nature given in Table 3. Neverthless, when considering cases involving related persons, the expected utilities are more sensitive to utility values, but the optimal decision is to type 9 markers in each case (see Fig. 6, right).

For a judge, the optimal decision is not very sensitive to changes in utilities (see Fig. 7, right).

In conclusion, the optimal decision is sensitive to strong changes in utilities for the prosecutor (decreasing the number of analysis to perform in case (*c*)). For the defense lawyer and the judge, different choices, in terms of utility values for the states of nature, affect the expected utilities for a few number of markers, but they have a very slight effect on its maximum, which represents the optimal number of markers to type.

### *Sensitivity to Prior Probabilities*

A simple result that follows from Bayes' theorem is that it is inadvisable to attach a probability equal to zero for states of nature; if the prior probability is zero, so is the posterior, whatever the data. In other words, if a decision-maker thinks something cannot be true, he will never be influenced by any data. A probability of one is equally dangerous (9). According to this, $\Pr(H_p)$ and $\Pr(H_d)$ should not be imperatively fixed at 1 or 0.

Sensitivity analysis is performed to check how different prior probabilities affect the optimal decision. Figure 8 represents the BDN used to perform sensitivity analysis to probabilities for the states of nature. No sensitivity analysis has been performed for the judge's perspective, because of its neutral position (equal prior probabilities for $\Pr(H_p)$ and $\Pr(H_d)$).

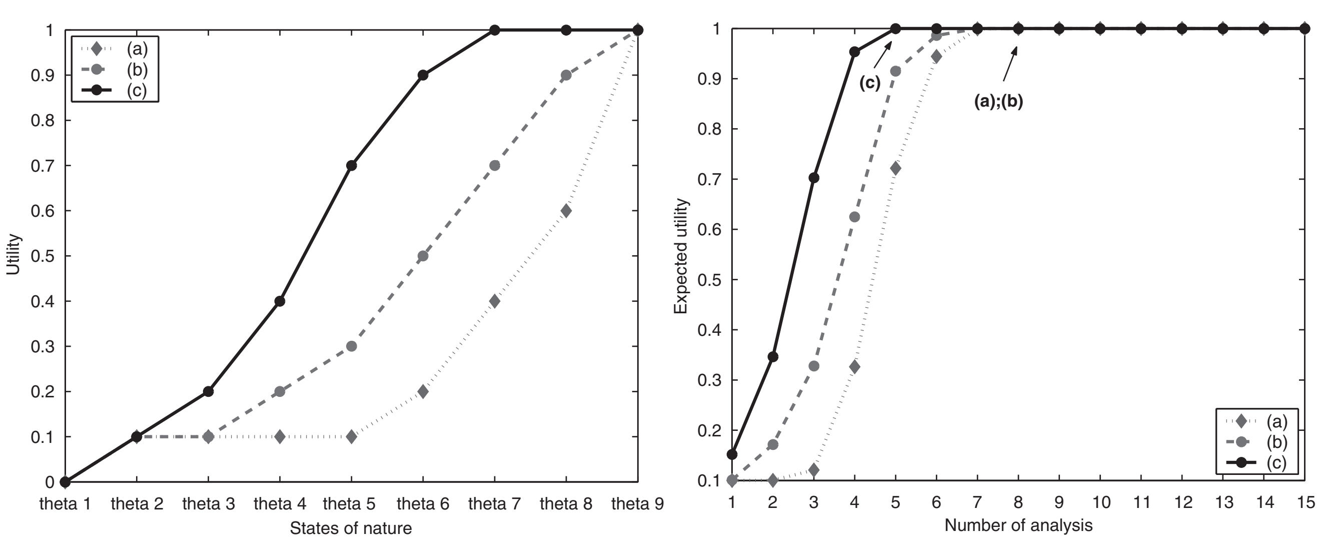


FIG. 5—*Prosecutor's perspective: Utility functions (left). Sensitivity of the optimal decision to the utility values (right). The arrows show the optimal decision.*

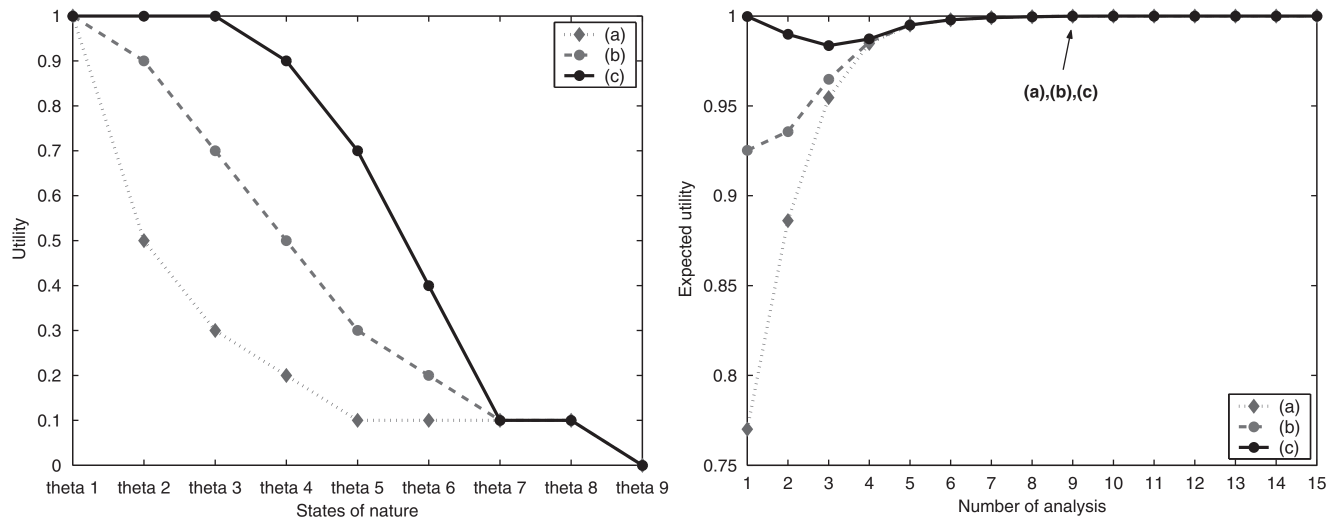


FIG. 6—*Defense lawyer's perspective: Utility functions (left). Sensitivity of the optimal decision to the utility values when the alternative proposition $H_d$ includes full siblings only (right). The arrow shows the optimal decision.*

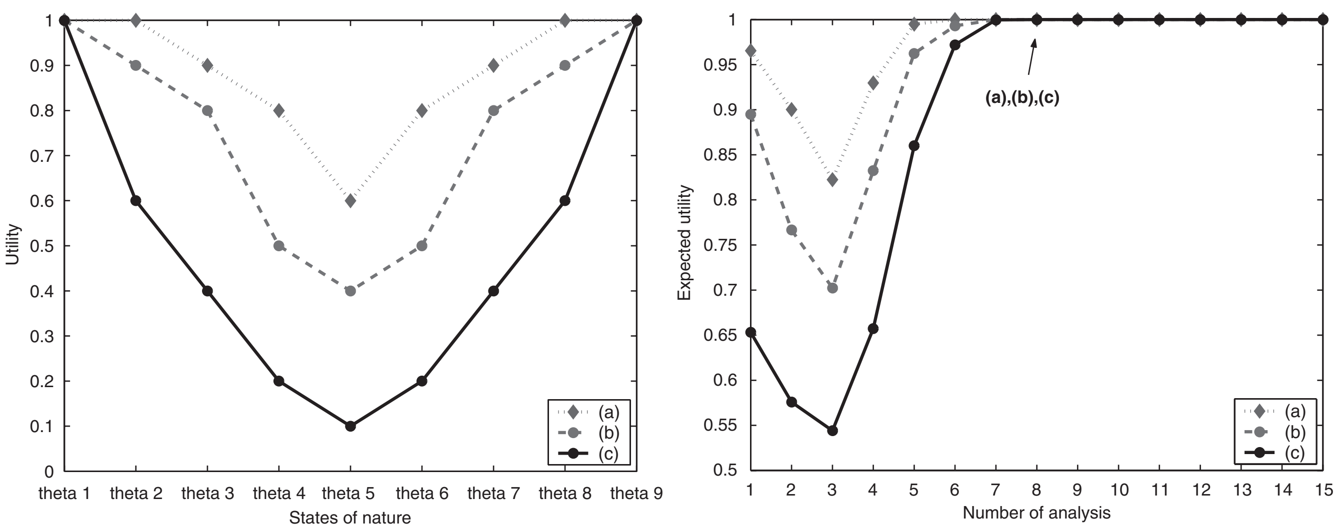


FIG. 7—*Judge's perspective: Utility functions (left). Sensitivity of the optimal decision to the utility values (right). The arrow shows the optimal decision.*

The prior probabilities, $\Pr(H_p)$ for the prosecutor and $\Pr(H_d)$ for the defense, have been modified from 1 to 0.6. These changes have no consequences on the final decision (see Table 8 and 9. Note that expected utilities for $d_8, \ldots, d_{15}$ are constant).

From Table 8, it is observed that prior probabilities $<1$ do not affect the optimal number of markers to type, although the expected utilities are lower. From Table 9, it can be supported that the greater the doubt about the fact that the suspect is or is not the source of the stain, the smaller the expected utilities will be (see the values in the first line in Table 9). Moreover, with $\Pr(H_p<1)$, expected utilities decrease as the total number of markers typed increases (see values in the second to fifth column in Table 9); therefore, the optimal decision will clearly be to type no more than a single marker to avoid a possible match between DNA profiles coming from the suspect and the crime stain.

## Discussion and Conclusion

Elements suggesting a solution to the question of interest, how many markers must I type to compare the crime sample with the suspect's material for forensic purposes? have been presented.

Decision analysis has been introduced to deal with this problem and it has been advocated that this kind of analysis helps the scientist to better understand the problem he is faced with and to make clearer and more consistent decisions (i.e., why is a chosen action appropriate?).

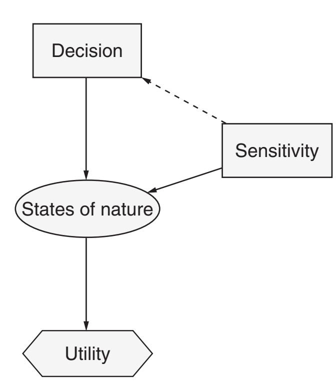


FIG. 8—*Influence diagram to perform the sensitivity analysis to the probabilities for the states of nature.*

TABLE 8—*Prosecutor's perspective: sensitivity of the expected utility to prior probabilities.*

| | $Pr(H_p) = 1$ | $Pr(H_p) = 0.9$ | $Pr(H_p) = 0.8$ | $Pr(H_p) = 0.7$ | $Pr(H_p) = 0.6$ |
|---|---|---|---|---|---|
| 1 locus | 0.1 | 0.09087 | 0.08175 | 0.07263 | 0.06351 |
| 2 loci | 0.1022 | 0.09026 | 0.08031 | 0.07036 | 0.06041 |
| 3 loci | 0.17818 | 0.16037 | 0.14255 | 0.12474 | 0.10692 |
| 4 loci | 0.66907 | 0.60216 | 0.53526 | 0.46835 | 0.40144 |
| 5 loci | 0.92490 | 0.83241 | 0.73992 | 0.64743 | 0.55494 |
| 6 loci | 0.98585 | 0.88726 | 0.78868 | 0.69009 | 0.59151 |
| 7 loci | 0.99978 | 0.89980 | 0.79983 | 0.69985 | 0.59987 |
| 8 loci | 1 | 0.9 | 0.8 | 0.7 | 0.6 |

TABLE 9—*Defense's perspective: sensitivity of the expected utility to prior probabilities.*

| | $Pr(H_p) = 1$ | $Pr(H_p) = 0.9$ | $Pr(H_p) = 0.8$ | $Pr(H_p) = 0.7$ | $Pr(H_p) = 0.6$ |
|---|---|---|---|---|---|
| 1 locus | 0.9861 | 0.9672 | 0.9483 | 0.9295 | 0.9106 |
| 2 loci | 0.9970 | 0.9529 | 0.9089 | 0.8649 | 0.8209 |
| 3 loci | 0.9998 | 0.9302 | 0.8606 | 0.7910 | 0.7215 |
| 4 loci | 1 | 0.9146 | 0.8292 | 0.7437 | 0.6583 |
| 5 loci | 1 | 0.9064 | 0.8129 | 0.7193 | 0.6258 |
| 6 loci | 1 | 0.9014 | 0.8028 | 0.7042 | 0.6056 |
| 7 loci | 1 | 0.9 | 0.8 | 0.7001 | 0.6001 |
| 8 loci | 1 | 0.9 | 0.8 | 0.7 | 0.6 |

Utility values have been used; they represent a personal measure of objective achievement. Utility as a measure of the value of the information (a measure of the benefit given by every new marker typed) has also been developed. It has been shown that in a scenario involving full siblings as a relevant population (the more extreme situation in favor of the defense), from the ninth marker on, the contribution of any new marker is extremely low, notably $<0.05\%$. This means that every new marker will add very little to the overall information already available.

Utility values used in decision analysis have been assumed constant for all possible decisions. This assumption can be relaxed; a decision-maker can specify that the same piece of information (for example, an apportionment of the likelihood ratio) does not have the same value if it is obtained after 1 marker is typed or after 15 markers have been typed. This situation appears in a transparent way using graphical models. Here, in fact, there are arcs from "States of Nature" and "Decision" nodes to "Utility" nodes capturing the idea that a scientist's satisfaction will depend on a combination of a result and an action. The preferences are made explicit in the (conditional) utility table.

In conclusion, it can be claimed that a good decision is one that makes effective use of the information available to the decision-maker at the time the decision is made. The specific information the scientist has, or can obtain, will help him to decide coherently among actions. Expected benefit is a way to do it. It has been shown that an arbitrary increase in the number of markers in DNA typing does not seem to be supported by decision analysis.

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